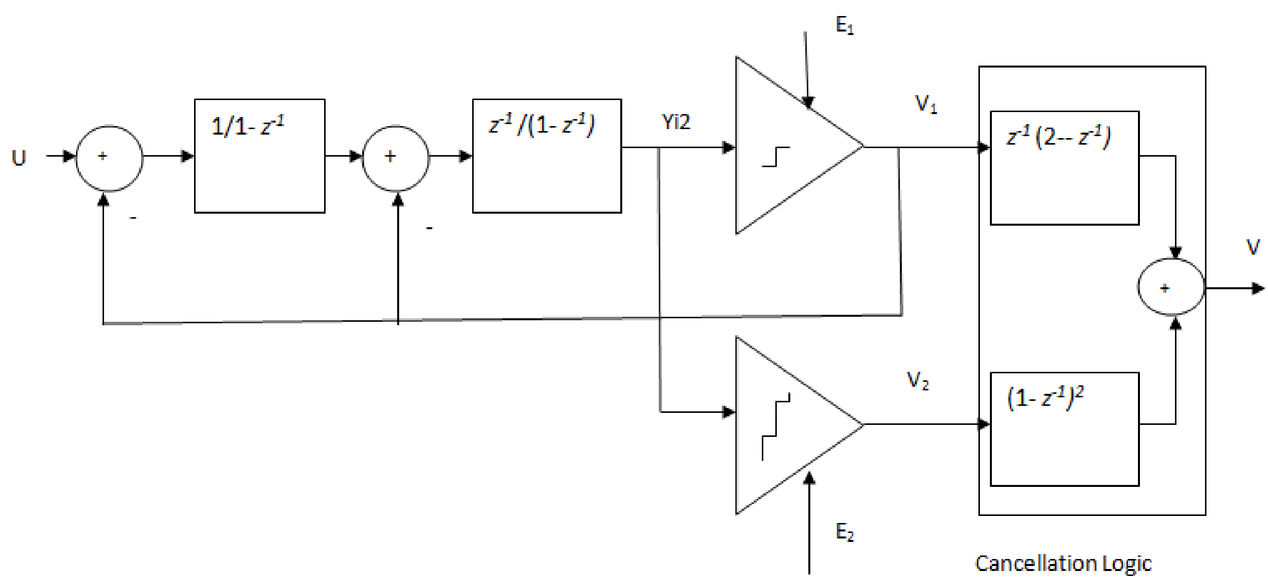
Problem3:(5 points) A Cascade Delta-Sigma ADC is presented in Figure 3.

(a) Derive the expressions for $V_1(z)$ and $V_2(z)$ in terms of U, E_1 and E_2 for the system.

(b) Show that if the cancellation logic implements

$V(z)=(2*z^{-1}- z^{-2}).V_1(z)+ (1- z^{-1})^2. V_2(z)$

then the output V(z) consists of the (essentially) unfiltered input U(z) plus the second order shaped $E_2(z)$ and the quantization noise of the one-bit quantizer $E_1(z)$ has been cancelled.



(a) From the block diagram,

$$((U- V_1/(1- z^{-1}) - V_1) (z^{-1} /(1- z^{-1})) + E_1 = V_1$$

Solving,

$$z^{-1} .U + (1- z^{-1})^2. E_1 = (1-2 z^{-1} + z^{-2} + z^{-1} + z^{-1} - z^{-2}). V_1$$

$$V_1(z) = z^{-1} . U(z) + (1 - z^{-1})^2 . E_1(z)$$

Also, $V_2 = Y_{i2} + E_2$

Solving,

$$V_2 = ((U - V_1)/(1 - z^{-1}) - V_1) . (z^{-1} / (1 - z^{-1})) + E_2$$

$$V_2 = z^{-1} / (1 - z^{-1})^2 . U - (2 - z^{-1}) z^{-1} / (1 - z^{-1})^2 . V_1 + E_2 \quad \text{........(1)}$$

$$V_2 = (z^{-1} - 2 z^{-2} + z^{-3}) / (1 - z^{-1})^2 . U + z^{-1} (2 - z^{-1}) . E_1 + E_2$$

$$V_2(z) = z^{-1} . U(z) + z^{-1} (2 - z^{-1}) . E_1(z) + E_2(z)$$

(b) If the cancellation logic is implemented, then

$$V = (2z^{-1} - z^{-2}) . V_1 + (1 - z^{-1})^2 . V_2$$

Substituting for V_2 from (1),

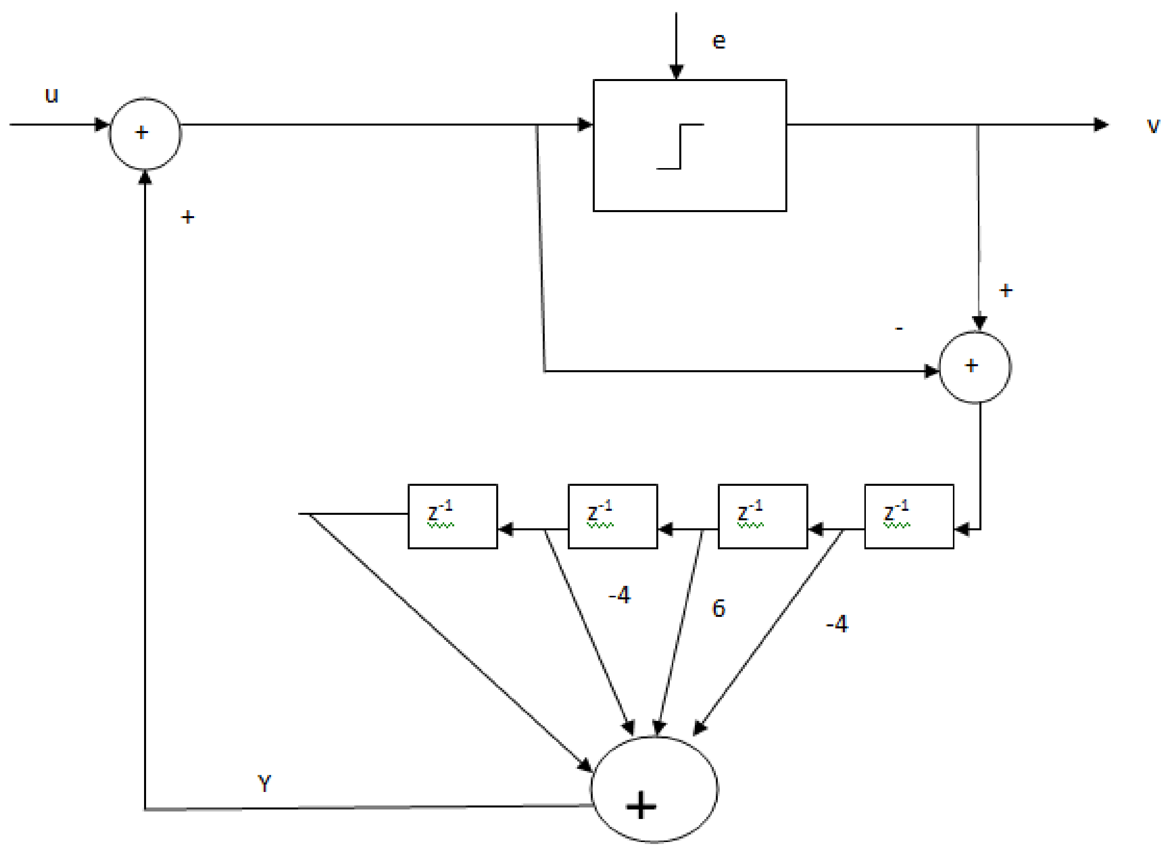
$$= (2z^{-1} - z^{-2}) . V_1 + (1 - z^{-1})^2 [z^{-1} / (1 - z^{-1})^2 . U - (2 - z^{-1}) z^{-1} / (1 - z^{-1})^2 . V_1 + E_2]$$

$$V(z) = z^{-1} . U(z) (1 - z^{-1})^2 . E_2(z)$$

Problem3: (10 points) A Delta-Sigma ADC is presented in figure 3.

(a) Derive the expression for V(z) in function of the input signal U(z) and the quantization noise E(z) and find the signal transfer function STF(z) and the noise transfer function NTF(z)

(b) Given that the Signal-to-noise-ratio SNR is -22.5 dB for OSR=1, what is the SNR when OSR changes up to 16?



(a) $V(z) = U(z)+Y(z)+E(z)$

Also, from the figure,

$Y(z) = E(z) [-4 z^{-1}+6 z^{-2}-4 z^{-3}+ z^{-4}]$

$V(z) = U(z) + E(z) [1 -4 z^{-1}+6 z^{-2}-4 z^{-3}+ z^{-4}]$

$V(z)= U(z) + E(z) (1- z^{-1})^4$

$STF(z) = 1$, $NTF(z) = (1- z^{-1})^4$

(b) Since this is a fourth order modulator, the improvement in SNR is given by (20L+10) log(OSR) where L is the order of the modulator. Hence, the improvement in this case is obtained using L=4, OSR=16.

$SNR= -22.5 + (20L+10) \log(OSR) = -22.5+ 90\log(16)$

SNR= 85.9 dB